Biorthogonal polynomials and nonautonomous discrete integrable systems

Kazuki Maeda
Department of Mathematical Sciences, Kwansei Gakuin University, Japan
Email: kmaeda@kmaeda.net

There are several known connections between integrable systems and orthogonal functions. For example, the three-term recurrence relation and spectral transformation of orthogonal polynomials give the Lax pair of the nonautonomous discrete Toda (nd-Toda) lattice

$$q_n^{(t+1)} + e_{n-1}^{(t+1)} + s^{(t+1)} = q_n^{(t)} + e_n^{(t)} + s^{(t)},$$
$$q_n^{(t+1)} e_n^{(t+1)} = q_{n+1}^{(t)} e_n^{(t)}.$$

One can construct a solution to the above difference equation under the semi-infinite or finite lattice condition by using determinants whose entries are the moments of linear functionals.

In this talk, based on the recent paper (arXiv:1703.06709 [nlin.SI]), we discuss a generalization of the connection above. It is shown that a nonautonomous version of the discrete two-dimensional Toda lattice is derived as the compatibility condition of the spectral transformations of biorthogonal polynomials. One can construct its solution in the same manner. Furthermore, imposing reduction conditions yield many nonautonomous discrete Toda-type systems including the nd-Toda lattice and the nonautonomous discrete hungry Toda (ndh-Toda) lattice. We derive a solution to the ndh-Toda lattice and consider its applications.